

# Cascade AOA Estimation Based on Combined Array Antenna with URFA and UCA

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**Abstract**— Since most studies for estimating an angle-of-arrival (AOA) based on the antenna array have considered the antenna array with a single configuration, they are not proper to simultaneously estimate AOAs of multiple signals with various frequencies. In order to enhance this problem, in this paper, we propose a cascade AOA estimation technique based on a Combined Array Antenna (CAA) with Uniform Rectangular Frame Array (URFA) and Uniform Circular Array (UCA). It consists of Capon for roughly finding AOA groups including multiple signal AOAs, followed by Beamspace Multiple Signal Classification (MUSIC) for detailedly estimating signal AOAs in the calculated AOA groups. The proposed algorithm does not only have low computational complexity compared to the conventional AOA estimation technique like MUSIC, but also it has both characteristics of URFA and UCA.

**Keywords**— *combined array antenna; AOA estimation; cascade estimation; Capon; Beamspace MUSIC*

## I. INTRODUCTION

The estimation algorithm of a signal AOA is not only one of core techniques in the location estimation, but also it has a variety of applications in the modern wireless communication systems. A representative AOA estimation algorithm with the high resolution is the MUSIC algorithm based on the eigenvalue decomposition of the covariance matrix of the received signal, which may employ various types of antenna array [1-3]. However, if it uses the antenna array with a single configuration, it might have the limitation in simultaneously estimating AOAs of multiple signals with various frequencies.

In this paper, we propose a cascade AOA estimation algorithm based on CAA structure, for efficiently estimating multiple signal AOAs with various frequencies. The inner array of CAA employs a URFA suitable to the narrow space, and the outer array of CAA employs a UCA with the large antenna element efficient to the low frequency signal. The configuration of CAA considers the efficient arrangement of URFA and UCA, in the limited space. The cascade algorithm consists of Capon and Beamspace MUSIC. Capon roughly finds the AOA groups including multiple signal AOAs, using only UCA among entire elements, and Beamspace MUSIC detailedly estimates signal AOAs in ranges calculated from AOA groups, using entire antenna elements. Since the proposed algorithm searches angles in only estimated ranges, it

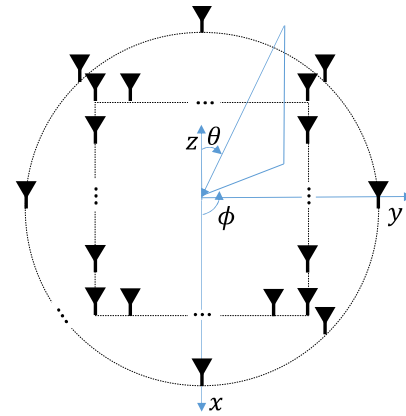


Fig. 1. Geometry of combined array antenna

has low computational complexity compared to the conventional AOA estimation techniques.

## II. RECEIVED SIGNAL MODEL

Assuming that  $L$  signals are incident on an antenna array with the CAA structure shown in Fig. 1, consisting of URFA and UCA, the received signal vector at the discrete sample index  $k$  can be modeled as

$$\mathbf{r}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k) \quad (1)$$

where  $\mathbf{A}$  is an array manifold matrix (size  $M \times L$ ),  $\mathbf{s}(k)$  is a signal vector (size  $L$ ), and  $\mathbf{n}(k)$  is an additive white Gaussian noise (AWGN) vector (size  $M$ ) with independent and identically distributed (i.i.d) components (zero mean and variance  $\sigma^2$ ). The array manifold matrix  $\mathbf{A}$  in (1) is defined as

$$\mathbf{A} = [\mathbf{A}_r \quad \mathbf{A}_c]^T \quad (2)$$

where  $\mathbf{A}_r$  and  $\mathbf{A}_c$  are URFA and UCA manifold matrices, respectively, described in [4][5]. Each column of  $\mathbf{A}$  represents the array response vector for each signal.

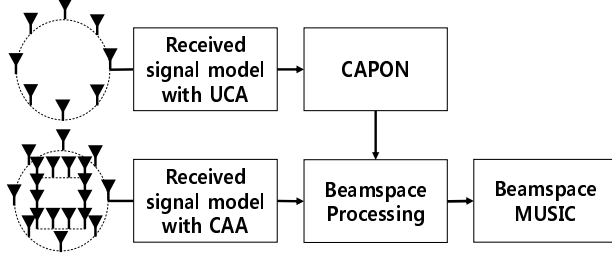


Fig. 2. Architecture of the proposed cascade AOA estimation algorithm

### III. CASCADE AOA ESTIMATION BASED ON CAA

In this section, we introduce the cascade AOA estimation technique based on CAA, with the excellent performance for simultaneously estimating various signals. It is made up of Capon for finding the approximate AOA groups, followed by Beamspace MUSIC for estimating the detailed signal AOAs in the estimated AOA groups, shown in Fig. 2.

#### A. Capon for AOA Group

Since the goal of Capon in this paper is to roughly find AOA groups including multiple signal AOAs, it utilizes the received signal based on only UCA among entire antenna elements, resulting in the low computational complexity. The Capon spatial spectrum based on the UCA received signal is defined as

$$P_{\text{CAPON}} = \frac{1}{\mathbf{a}(\theta, \phi)^H \mathbf{R}_{\text{UCA}}^{-1} \mathbf{a}(\theta, \phi)}, \quad (3)$$

where  $\mathbf{R}_{\text{UCA}}^{-1} = E[\mathbf{r}_{\text{UCA}}(k) \mathbf{r}_{\text{UCA}}^H(k)]$  is the covariance matrix for the received signal incident on UCA,  $\mathbf{r}_{\text{UCA}}(k)$  is the received signal vector based on UCA,  $\mathbf{a}(\theta, \phi)$  is the array response vector for specific elevation and azimuth angles, and  $(\cdot)^H$  is the conjugate transpose. Using peaks of (3), we roughly estimate AOA groups including multiple signal AOAs. From AOA groups found by Capon, we calculate ranges of existing signals.

#### B. Beamspace MUSIC for Detailed Signal AOA

Since the goal of Beamspace MUSIC in this paper is to estimate the detailed signal AOAs in ranges calculated by Capon, it utilizes entire elements of CAA. The Beamspace MUSIC spatial spectrum for the  $i$ th AOA group is defined as

$$P_{\text{BMUSIC}(i)} = \frac{1}{\left[ \mathbf{B}_{(i)} \mathbf{a}(\theta, \phi) \right]^H \mathbf{E}_{n(i)} \mathbf{E}_{n(i)}^H \left[ \mathbf{B}_{(i)} \mathbf{a}(\theta, \phi) \right]}, \quad (4)$$

where  $\mathbf{B}_{(i)}$  is the beamspace transformation matrix for the  $i$ th AOA group, presented in [6][7], and  $\mathbf{E}_{n(i)}$  is the beamspace noise subspace, calculated from the eigenvalue decomposition of the beamspace covariance matrix for the  $i$ th AOA group defined as

$$\mathbf{R}_{B(i)} = E \left[ \mathbf{B}_{(i)} \mathbf{r}(k) \mathbf{r}^H(k) \mathbf{B}_{(i)}^H \right]. \quad (5)$$

Using peak values of (4), we estimate the individual signal AOAs included in the  $i$ th AOA group. In order to estimate all signal AOAs, this process is repeated for all found AOA groups. The proposed cascade AOA estimation technique has much lower computational complexity compared with the conventional AOA estimation algorithm like MUSIC, because it searches angles only in the calculated ranges.

### IV. COMPUTER SIMULATION

In this section, we provide computer simulation results to demonstrate the AOA estimation performance of the proposed technique. For this simulation, we assume that URFA with 12 elements and UCA with 16 elements are employed in CAA, and the received signal includes one amplitude modulation (AM) signal, two continuous wave (CW) signals, two wideband (WB) noise signals, and AWGN. In addition, parameters of each signal are summarized in Table 1 and the signal-to-noise ratio (SNR) for each is assumed to be 20 dB.

Fig. 3 shows the spectrum of the received signal, that we observe five signals located at the normalized frequencies. Fig. 4 shows the spatial spectrum of Capon based on UCA, and three AOA groups are identified in the figure. The spatial spectrum of Beamspace MUSIC based on entire antenna elements is shown in Fig. 5. From this figure, we observe that the AM and the first CW signals are estimated in the first AOA group, the second CW and the first WB noise signals are estimated in the second AOA group, and the second WB noise signal is estimated in the third AOA group. From these results, we confirm that the proposed AOA estimation algorithm based on CAA has good estimation performance.

TABLE I. SIGNAL PARAMETER

| Signal | Elevation | Azimuth  | Normalized frequency | Modulation index |
|--------|-----------|----------|----------------------|------------------|
| AM     | 60        | -165     | 0.08                 | 0.03             |
| CW     | 60, 60    | -161, 10 | 0.2, 0.4             | -                |
| WB     | 60, 60    | 17, 152  | 0.1, 0.3             | -                |

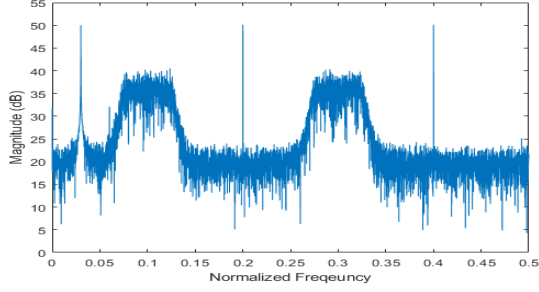


Fig. 3. Received signal spectrum

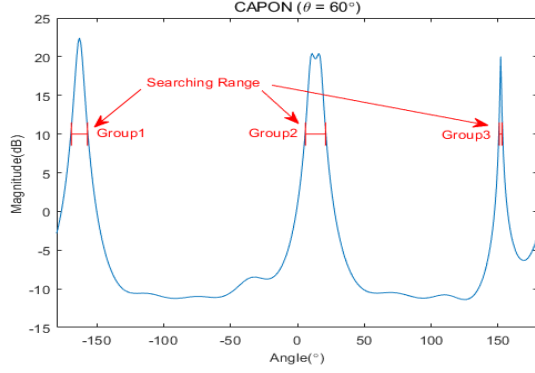


Fig. 4. Capon spatial spectrum based on UCA

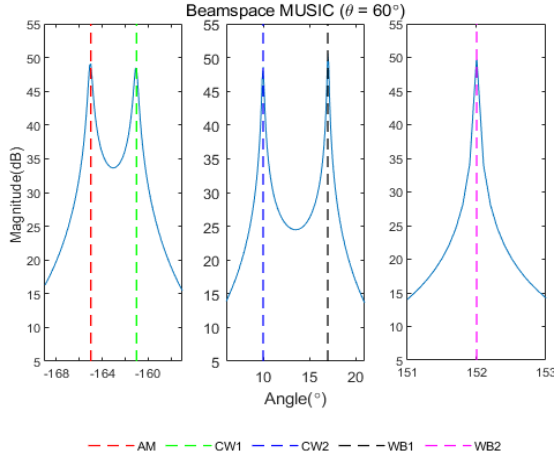


Fig. 5. Beamspace MUSIC spatial spectrum based on CAA

## V. CONCLUSION

In this paper, we proposed the cascade AOA estimation technique based on CAA consisting of URFA and UCA. The CAA structure is efficient for simultaneously estimating AOA of multiple signals with various frequencies, compared to the single configuration antenna array. The proposed cascade algorithm consists of Capon for finding AOA groups and Beamspace MUSIC for estimating individual signal AOA. In this paper, Capon utilizes only UCA among entire antenna elements, because it requires roughly finding AOA groups. On the other hand, Beamspace MUSIC utilizes entire antenna elements, because it requires estimating individual signal AOA in detail, in the ranges estimated by Capon. Since the proposed technique searches angles within only the estimated ranges, it has low computation complexity comparing to the conventional AOA estimator like MUSIC. In addition, we provided a computer simulation example for confirming the estimation performance of the proposed technique.

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